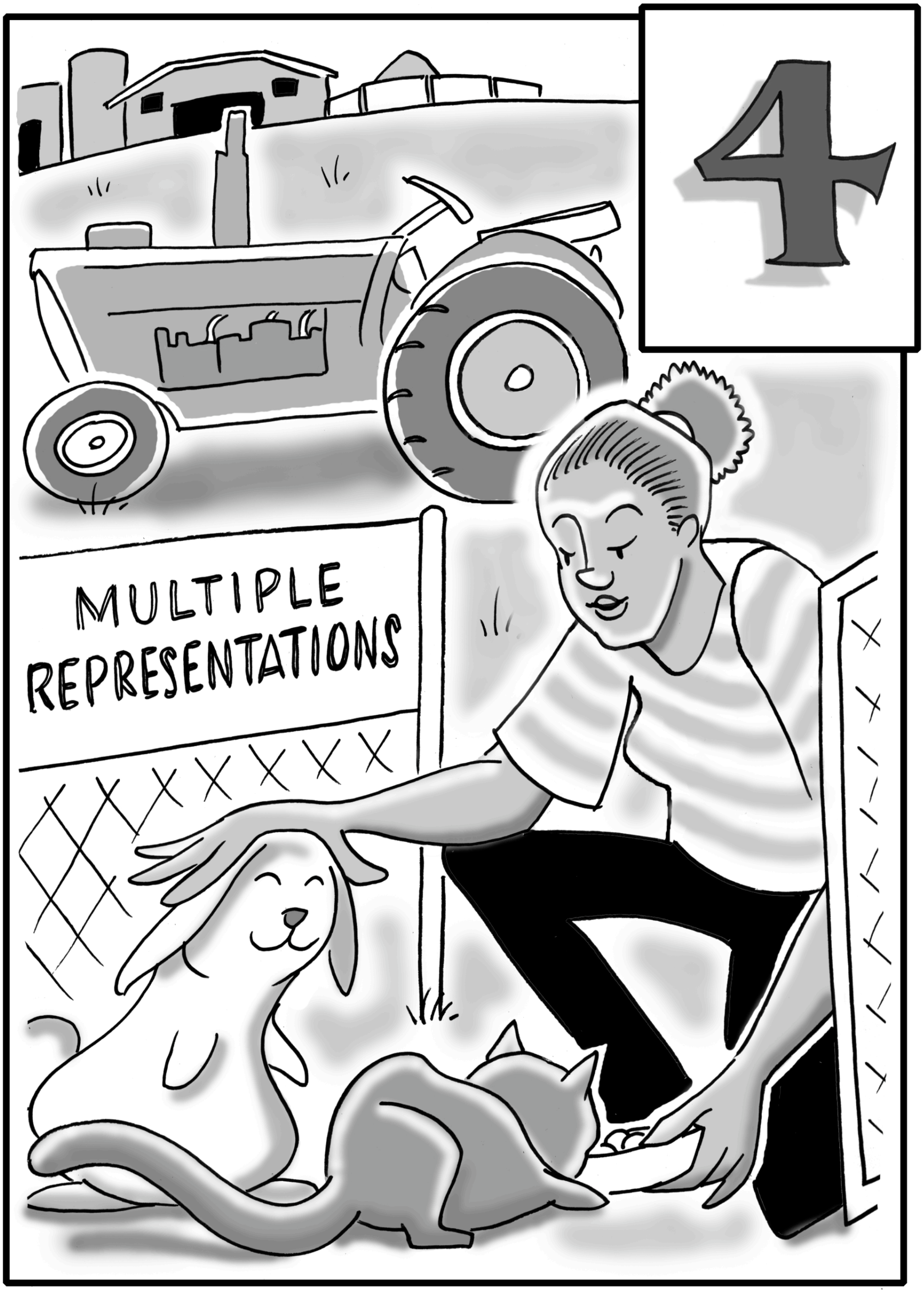

4
MULTIPLE
REPRESENTATIONS

CHAPTER 4 Multiple Representations

This chapter builds on the work you did in Chapters 2 and 3. The primary focus of Chapter 4 is to investigate the **connections** between the four representations of data: graphs, tables, patterns, and equations (also referred to as “rules”). You will also explore situations that can be represented by a line and study what it means when two lines intersect (cross each other). By the end of this chapter, you will know how to use graphs, tables, patterns, and rules to solve almost any problem involving lines.

Guiding Questions

Think about these questions throughout this chapter:

What is the connection?

Is there a pattern?

How many different ways can it be represented?

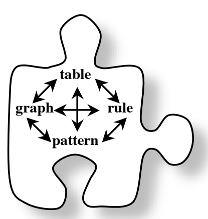
How does the pattern show up in the rule, table, and graph?

How does the pattern grow?

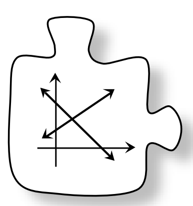
In this chapter, you will learn:

- How to change any representation of data (such as a graph, pattern, rule, or table) to any of the other representations.
- How to write an equation from a word problem.
- How to find the point where two lines intersect.
- How to use the connections between graphs, tables, rules, and patterns to solve problems.

Chapter Outline

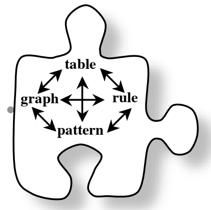


Section 4.1 In this section, you will shift between different representations of linear patterns by using the web shown at left. By finding connections between each representation, you and your team will find ways to change from one representation to each of the other three representations.



Section 4.2 Section 4.2 will start by examining word problems in which two amounts are compared. You will use your knowledge of graphs and rules to write equations for word problems. Then, using the equation mat, you will solve a linear equation to determine where two lines cross. A final challenge will bring together word problems and the representations in the web.

4.1.1 What's the connection?

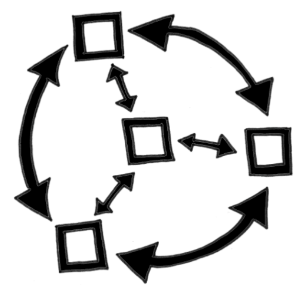


Finding Connections Between Representations

In Chapter 3 you studied different ways to represent patterns. You organized information into tables, graphed information about patterns, and learned how to find the rules that govern specific patterns.

Starting today and continuing throughout this chapter, you will find **connections** between different representations of the same pattern, explore each representation more deeply, and learn shorter ways to go from one representation to another. By the end of this chapter, you will have a deeper understanding of many of the most powerful tools of algebra.

4-1. TILE PATTERN TEAM CHALLENGE



Your teacher will assign your team a tile pattern (one of the patterns labeled (a) through (e) on the next page). Your team's task is to create a poster showing every way you can represent your pattern and highlighting all of the connections between the representations that you can find. For this activity, **finding and showing the connections are the most important parts.** Clearly presenting the connections between representations on your poster will help you convince your classmates that your description of the pattern makes sense.

Pattern Analysis:

- **Extend** the pattern: Draw Figures 0, 4, and 5. Then describe Figure 100. Give as much information as you can. What will it look like? How will the tiles be arranged? How many tiles will it have?
- **Generalize** the pattern by writing a rule that will give the number of tiles in any figure in the pattern. Show how you got your answer.
- Find the number of tiles in each figure. Record your data in a table and on a graph.
- Demonstrate how the pattern grows using color, arrows, labels, and other math tools to help you show and explain. Show growth in each representation.
- What **connections** do you see between the different representations (graph, figures, and $x \rightarrow y$ table)? How can you show these **connections**?

Problem continues on next page →

4-1. *Problem continued from previous page.*

Presenting the Connections:

As a team, organize your work into a large poster that clearly shows each representation of your pattern, as well as a description of Figure 100. When your team presents your poster to the class, you will need to support each statement with a reason from your observations. Each team member must explain something mathematical as part of your presentation.

a.

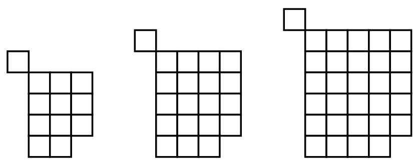


Figure 1 Figure 2 Figure 3

b.

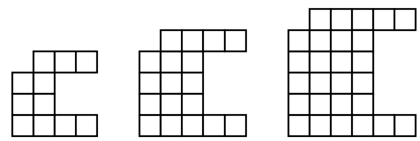


Figure 1 Figure 2 Figure 3

c.

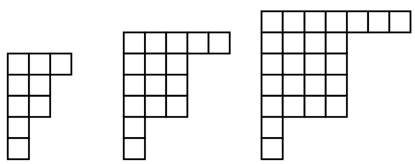


Figure 1 Figure 2 Figure 3

d.

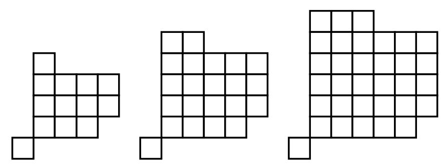


Figure 1 Figure 2 Figure 3

e.

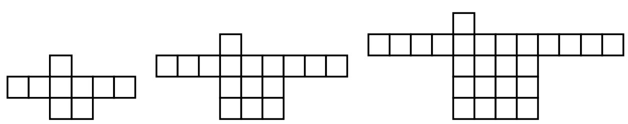


Figure 1 Figure 2 Figure 3

Review & Preview

4-2. For each tile pattern in problem 4-1, draw Figures 0, 4, and 5 on graph paper. If it helps, copy Figures 1, 2, and 3 onto your paper.

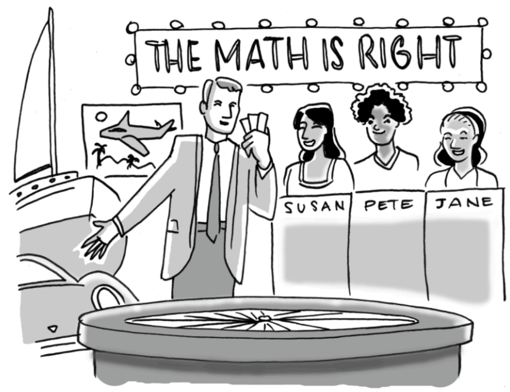
4-3. Make an $x \rightarrow y$ table for the rule $y = x^2 - 2x$.

a. Plot and connect the points on a complete graph.

b. Does your graph look like a full parabola? If not, add more points to your table and graph to complete the picture.

4-4. THE GAME SHOW

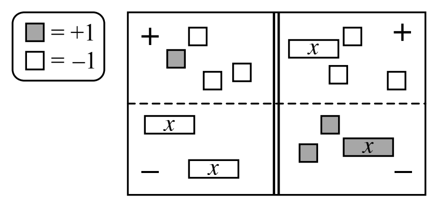
Susan had an incredible streak of good fortune as a guest on the exciting game show, "The Math Is Right." She amassed winnings of $12,500, a sports car, two round-trip airline tickets, and five pieces of furniture.



In an amazing finish, Susan then landed on a "Double Your Prize" square and answered the corresponding math question correctly! She instantly became the show's biggest winner ever, earning twice the amounts of all her previous prizes.

A week later, $25,000, a sports car, four round-trip airline tickets, and five pieces of furniture arrived at her house. Susan felt cheated. What was wrong?

4-5. Write the equation represented by the diagram at right.



a. Simplify as much as possible and then solve for x.

b. Check your solution.

4-6. Copy and simplify the following expressions by combining like terms.

a. $y+3x-3+2x^2+8x-5y$

b. $2x+4x^2-6x^2-9+1-x-3x$

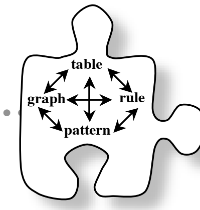
c. $2y^2+30y-3y^2+4y-14-y$

d. $-10+3xy-3xy+y^2+10-y^2$

4-7. Use your pattern-finding techniques to fill in the missing entries for the table below. Then find a rule for the pattern.

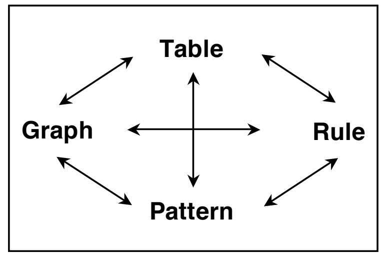
IN (x)	4	8	3	–2	–6	0	5	7
OUT (y)	17	65	10	5		1	26	

4.1.2 How does it grow?



Seeing Growth in Different Representations

In Lesson 4.1.1, you looked at four different ways of representing patterns and began to find **connections** between them.



Throughout this chapter you will explore **connections** and find shortcuts between the representations. Today, you will look for specific connections between geometric patterns and equations. As you work today, keep these questions in mind:

How can you see growth in the rule?

How do you know your rule is correct?

What does the representation tell you?

What are the connections between the representations?

At the end of this lesson, put your work from today in a safe place, because you will need to use it during Lesson 4.1.3!

4-8. **Tile Pattern #1:**

Examine the tile pattern at right.

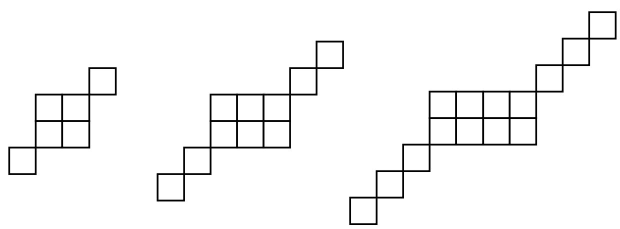


Figure 1 Figure 2 Figure 3

a. What do you notice? After everyone has had a moment on his or her own to examine the figures, discuss what you see with your team.

b. Sketch the next figure in the sequence (Figure 4) on your resource page. Sketch the figure that comes before Figure 1 (Figure 0).

c. How is the tile pattern growing? Where are the tiles being added with each new figure? Color in the new tiles in each figure with a marker or colored pencil on your resource page.

d. What would Figure 100 look like? Describe it in words. How many tiles would be in the 100^{th} figure? Find as many ways as you can to **justify** your conclusion. Be prepared to report back to the class with your team's findings and methods.

4-9. Answer questions (a) through (d) from problem 4-8 for each of the patterns below. Use color to shade in the new tiles on each pattern on your resource page. Choose one color for the new tiles in part (a) and a different color for the new tiles in part (b).

a. **Tile Pattern #2:**

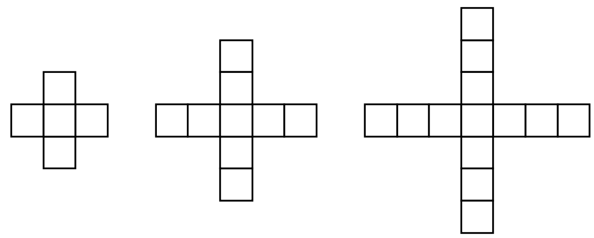


Figure 1 Figure 2 Figure 3

b. **Tile Pattern #3:**

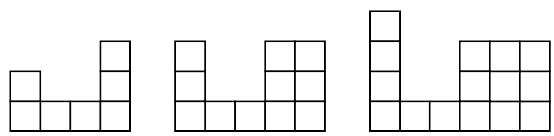


Figure 1 Figure 2 Figure 3

4-10. PUTTING IT TOGETHER

Look back at the three different tile patterns in problems 4-8 and 4-9 to answer these questions.

a. What is the same and what is different between these three patterns? Explain in a few sentences.

b. Write an equation (rule) for the number of tiles in each pattern.

c. What connections do you see between your equations and the tile pattern? Show and explain these connections.

d. Imagine that the team next to you created a new tile pattern that grows in the same way as the ones you have just worked with, but they refused to show it to you. What other information would you need in order to predict the number of tiles in Figure 100? Explain your reasoning.

4-11. Consider **Tile Pattern #4**, shown below.

a. Draw Figures 0 and 4 on the resource page.

b. Write an equation (rule) for the number of tiles in this pattern. Use a new color to show where the numbers in your rule appear in the tile pattern.

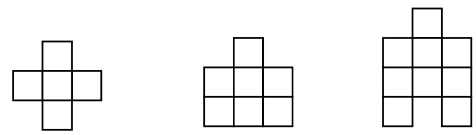


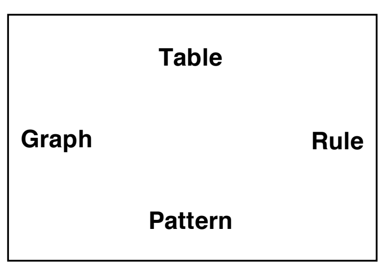
Figure 1 Figure 2 Figure 3

c. What is the same about this pattern and Tile Pattern #3? What is different? What do those similarities and differences look like in the tile pattern? In the equation?

d. How is the growth represented in each equation?

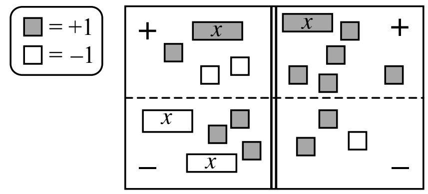
Don't forget to put your work from today in a safe place, because you will need to use it during the next lesson.

4-12. For today's Learning Log entry, draw a web of the different representations, starting with the diagram below. Draw lines and/or arrows to show which representations you have connected so far. Explain the connections you learned today. Be sure to include anything you figured out about how the numbers in equations (rules) relate to tile patterns. Title this entry "Starting the Web" and label it with today's date.



Review & Preview

4-13. Write the equation represented in the equation mat at right.



a. Simplify as much as possible and then solve for x.

b. Check your solution.

4-14. Simplify each of the following equations and solve for x. Show all work and check your solution.

a. $7-3x=-x+1$

b. $-2+3x=-(x+6)$

4-15. Leala can write a 500-word essay in an hour. If she writes an essay in 10 minutes, approximately how many words do you think the essay contains?

4-16. Copy and complete the table below.

IN (x)	2	10	6	7	–3		–10	1000	x
OUT (y)	9	25	17			15			

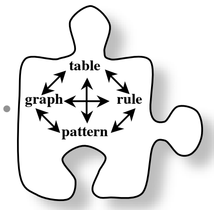
a. Explain in words what is done to the input value (x) to produce the output value (y).

b. Write the rule you described in part (a) with algebraic symbols.

4-17. When Susan's brother went to college, she and her two sisters evenly divided his belongings. Among his possessions were 3 posters, 216 books, and 24 CDs. How were these items divided?

4.1.3 How does it grow?

Connecting Linear Rules and Graphs



You have been looking at geometric patterns and ways that those patterns can be represented with equations, graphs, and $x \rightarrow y$ tables. In Lesson 4.1.2 you worked with four different tile patterns and looked for **connections** between the geometric shapes and the numbers in the equations. Today you will go back to those four equations and look for **connections** to other representations.

By the end of this lesson, you should be able to answer the following target questions:

How is growth shown in a rule?

How is growth shown in a graph?

How can you determine the number of tiles in Figure 0 from a graph?

How can you determine which tile pattern grows faster from a graph?

4-18. Examine your Lesson 4.1.2 Resource Page ("Pattern Analysis").

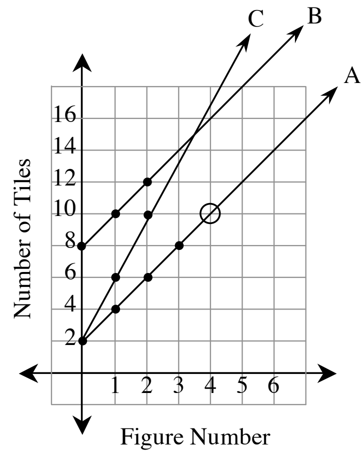
a. Make sure you have a rule for each tile pattern.

b. Draw a graph. Put all patterns on the same set of axes. Use different colors for each, matching the color you used on the resource page.

c. Explain how the growth appears in the pattern, in the rule, and in the graph.

d. What **connections** do you see between these representations? Describe any **connections** you see.

4-19. The graph at right is also on the Lesson 4.1.3 Resource Page provided by your teacher. It gives information about three new tile patterns. **Note:** In this course, tile patterns will be considered to be elements of continuous relationships and thus will be graphed with a continuous line or curve.



Answer the following questions as a team.

a. What information does the circled point (○) on the graph tell you about tile pattern A?

b. Find the growth of each tile pattern. For example, how much does tile pattern A increase from one figure to the next? Explain how you know.

c. Look at the lines for tile patterns A and B. What is the same about the two lines? What conclusion can you make about these tile patterns? What is different about the lines? What does this tell you about the tile patterns? **Justify** your answers, based on the graph.

d. Look at lines A and C on the graph. What do these two lines have in common? In what ways are the lines different? What does this tell you about the tile patterns? Explain completely.

4-20. In your Learning Log, answer the target questions for this lesson, reprinted below:

How is growth shown in a rule?

How is growth shown in a graph?

How can you determine the number of tiles in Figure 0 from a graph?

How can you determine which tile pattern grows faster from a graph?

Be sure to include at least one example. Title this entry "Connecting Linear Rules and Graphs" and label it with today's date.

Review & Preview

4-21. Two of the connections in your representations web are pattern → table and pattern → rule. Practice these connections as you answer the questions below.

a. On graph paper, draw Figure 0 and Figure 4 for the pattern at right.

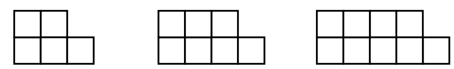


Figure 1   Figure 2   Figure 3

b. Represent the number of tiles in each figure with a table.

c. Represent the number of tiles in each figure with an algebraic rule.

4-22. For the rule $y = x^2 - 4$, calculate the y-values that complete the table below. Plot the points and connect them on a complete graph on graph paper. What does your graph look like?

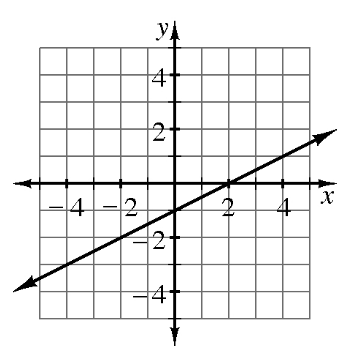
IN (x)	–3	–2	–1	0	1	2	3
OUT (y)							

4-23. For each of the equations below, solve for x. Show all work and check your solution.

a. $-2 + 2x = -x + 2 + x$ b. $2 - 3x = x + 2$

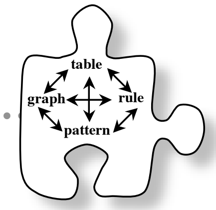
4-24. The length of a rectangle is five centimeters more than twice its width. The perimeter is 100 centimeters. Use Guess and Check to find out how long and how wide the rectangle is.

4-25. Another one of the connections in your representations web is graph → table. In Chapters 1 through 3, you developed tools to find a table from a graph. Consider this connection as you complete the table below based on the graph at right.



IN (x)	–3	–2	–1	0	1	2	3
OUT (y)							

4.1.4 What's the rule? How can I use it?



$y = mx + b$

In Lessons 4.1.2 and 4.1.3, you investigated connections between tile patterns, $x \rightarrow y$ tables, graphs, and rules (equations). Today you will use your observations about growth and Figure 0 to write rules for linear patterns and to create new tile patterns for given rules.

4-26. UNDERSTANDING $y = mx + b$

With your team, list some of the equations you have been working with in the past two lessons.

a. What do all of these rules have in common?

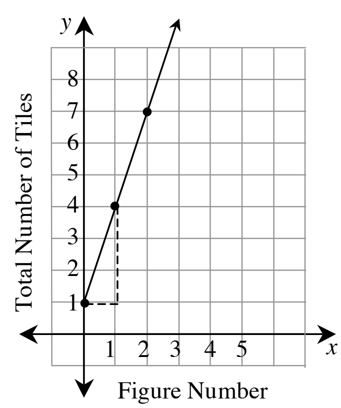
Rules for linear patterns can all be written in the form $y = mx + b$, where x and y represent variables, but m and b represent **constants** (numbers that stay the same in the equation after they are chosen). Discuss these questions with your team:

b. What does m tell you about the pattern?

c. What does b tell you about the pattern?

4-27. GRAPH $\rightarrow$ RULE

Allysha claims she can find the equation of a line by its graph without a table. How is that possible? Discuss this idea with your team and then try to find the equation of the line at right without first making a table. Be ready to share with the class how you found the rule.



4-28. TABLE → RULE

Allysha wonders if she can use the idea of m and b to find the equation of a line from its table.

a. For example, if she knows the information about a linear pattern given in the table below, how can she find the equation of the line? Work with your team to complete the table and find the rule.

IN (x)	0	1	2	3	4	5	6
OUT (y)	–2						

+5 +5 +5 +5 +5 +5

b. Use this same idea to find the rule of the linear tile patterns represented by the tables below.

i.

IN (x)	–1	0	1	2	3	4	5
OUT (y)	3	5	7	9	11	13	15

ii.

IN (x)	0	1	2	3	4	5	6
OUT (y)	7	4	1	–2	–5	–8	–11

c. Write a summary statement explaining how you used your knowledge about m and b to quickly write a rule.

4-29. RULE → PATTERN

In each problem below, invent your own pattern that meets the stated conditions. Draw Figures 0, 1, 2, and 3 and write the rule (equation) for your pattern.

a. A tile pattern that has $y = 4x + 3$ as a rule.

b. A tile pattern that decreases by 2 tiles and Figure 2 has 8 tiles.

4-30. Invent two different tile patterns that grow by 4 every time but have different $x \rightarrow y$ tables. Draw Figures 0, 1, 2, and 3 and find rules for each of your patterns. What is different about your rules? What is the same?

4-31. The linear equations you have been working with can be written in the general form:

$$y = mx + b$$

In your Learning Log, summarize what you know about m and b so far. What does the m tell you about a pattern? What does the b tell you about a pattern? Where can you see m and b in each representation? Sketch examples if it helps. Title this entry "$y = mx + b$" and label it with today's date.

Review & Preview

4-32. For each equation below, solve for x. Check your solution, if possible, and show all work.

a. $3x-6+1=-2x-5+5x$

b. $-2x-5=2-4x-(x-1)$

4-33. I am thinking of a number. When I double my number and then subtract the result from five, I get negative one. What is my number? Write and solve an equation.

4-34. Copy this table and use your pattern skills to complete it.

IN (x)	2	10				-3			x
OUT (y)	4	28	13	-17	10		2.5	148	$3x-2$

a. Explain in words what is done to the input value, x, to produce the output value, y.

b. Explain the process you used to find the missing input values.

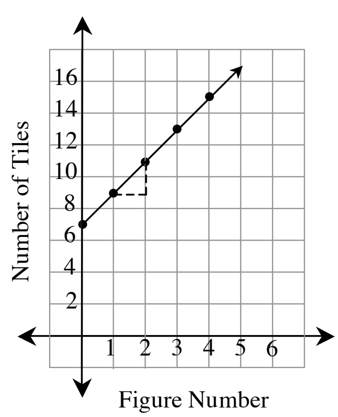
4-35. Examine the $x \rightarrow y$ table at right.

a. Invent a tile pattern that fits this data.

b. What is the pattern's growth factor? Show where the growth factor appears in the $x \rightarrow y$ table and the tile pattern.

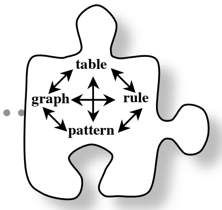
Figure Number	**Number of Tiles**
0	5
1	9
2	13
3	17

4-36.



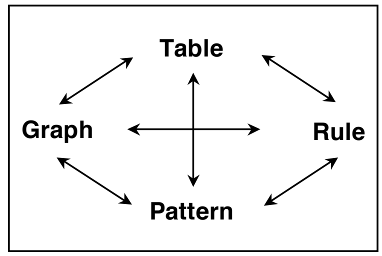
Look at the graph at left. What statements can you make about the tile pattern the graph represents? How many tiles are in Figure 0? Figure 1? What is the growth factor?

4.1.5 What are the connections?



Checking the Connections

In the last several lessons you have been finding **connections** and relationships between different representations of patterns. You have worked backward and forward and have used information about Figure 0 (or the starting point) and the growth factor in order to write rules. In today's activity, you will check your **connections** by using pieces of information from different parts of the web to generate a complete pattern.



4-37. CHECKING THE CONNECTIONS: TEAM CHALLENGE

Today you are going to **apply** what you know about the starting point (Figure 0), growth factor, and the **connections** between representations to answer some challenging questions. The information in each question, parts (a) through (d), describes a different pattern. The graph of each pattern is a line. From this information, generate the rule, $x \rightarrow y$ table, graph, and tile pattern (Figures 0 through 3) that follow the pattern. You may answer these questions in any order, but make sure you answer each one completely before starting another problem.

Work together as a team. The more you listen to how other people see the **connections** and the more you share your own ideas, the more you will know at the end of the lesson. Stick together and be sure to talk through every idea.

Each person will turn in his or her own paper at the end of this activity, showing four complete representations for each pattern. Your work does not need to be identical to your teammates' work, but you should have talked and agreed that all explanations are correct.

a.

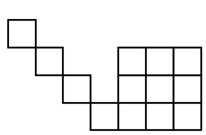
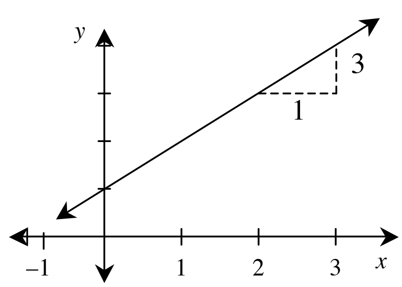


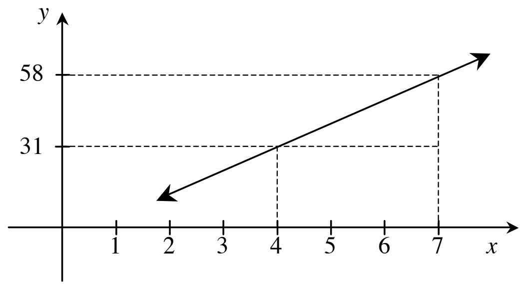
Figure 3



Problem continues on next page →

4-37. *Problem continued from previous page.*

b.



c.

Figure Number	Number of Tiles
0	
1	
2	
3	12

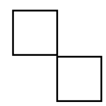
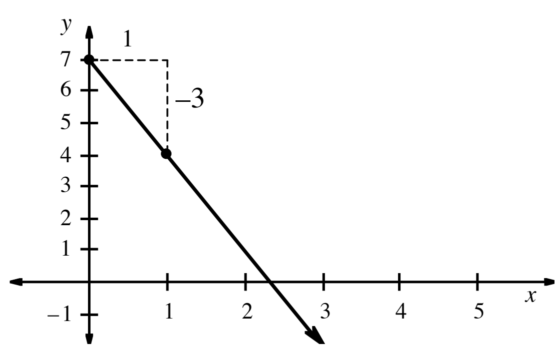


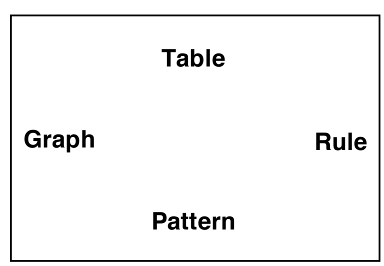
Figure 8

d. $y = -3x + 7$



4-38. REPRESENTATIONS WEB

Update your representations web from problem 4-12 with any new **connections** you have found. Pay attention to which direction(s) the arrow points.



Review & Preview

4-39. For each equation below, solve for the variable. Check your solutions, if possible, and show all work.

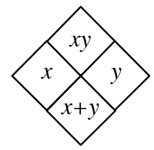
a. $-3 + x = -x + 5$

b. $-(x - 3) = 2x - 4 - 3x$

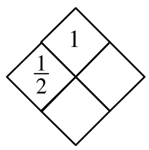
c. $2 + 4k = 2k + 9$

d. $-(-t + 4) = -3t$

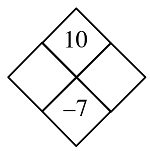
4-40. Copy and complete each of the Diamond Problems below. The pattern used in the Diamond Problems is shown at right.



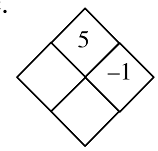
a.



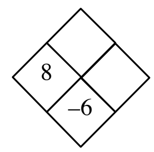
b.



c.



d.



4-41. Complete a table for the rule $y = 3x - 2$.

a. Draw a complete graph for this rule.

b. Is (–50, –152) a point on the graph? Explain how you know.

4-42. Write down everything you know about the tile pattern represented by the $x \to y$ table at right. Be as specific as possible.

x	y
3	25
5	39
6	46
1	11

4-43. Simplify each of the expressions below. You may use an equation mat and tiles.

a. $-(5x + 1)$

b. $6x - (-5x + 1)$

c. $-(1 - 5x)$

d. $-5x + (x - 1)$

4-44. Invent a tile pattern that grows by 4 each time. Draw Figures 0, 1, 2, and 3. Use color or shading to show the growth.

4-45. For each equation below, solve for the variable. Check your solutions, if possible, and show all work.

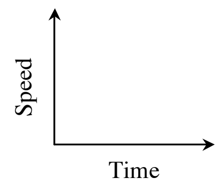
a. $3p - 7 + 9 - 2p = p + 2$

b. $-2x + 5 + (-x) - 5 = 0$

c. $12 = r + 6 - 2r$

d. $-(y^2 - 2) = y^2 - 5 - 2y^2$

4-46. Sketch a graph to match each story below using axes labeled as shown at right.



a. Luis rides his skateboard at the same speed all the way home. It takes him ten minutes to get there.

b. Corinna jogs along at the same speed until she reaches a hill, and then she slows down until she finally stops to rest.

c. Sergei is talking with his friends at the donut shop when he realizes that it's almost time for math class! He runs toward school, but slows to a walk when he hears the bell ring and realizes that he is already late. He sits down in class four minutes after he left the donut shop.

4-47. Complete a table for the rule $y = 3 - x$.

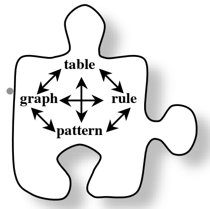
a. Draw a complete graph for this rule.

b. Is (32, –29) a point on this graph? Explain why or why not.

4-48. Mr. Wallis decided to create another table to figure out how much it costs to send a certain number of regular letters through the mail. Use proportional reasoning to complete his table at right.

Number of Letters	**Cost of Stamps**
10	\$3.40
2	\$0.68
	\$5.10
7	
1	
500	
	\$14.28

4.1.6 How can I use growth?

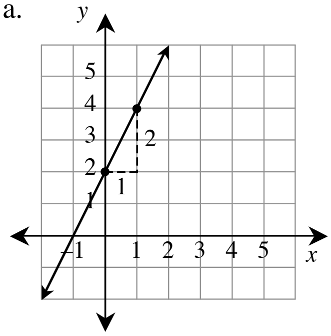


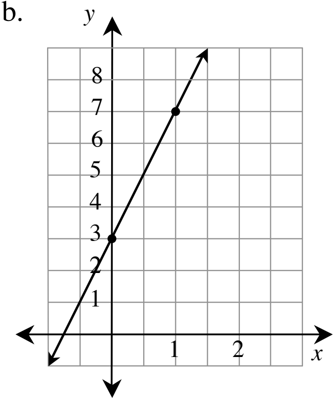
Graphing a Line Without an $x \rightarrow y$ Table

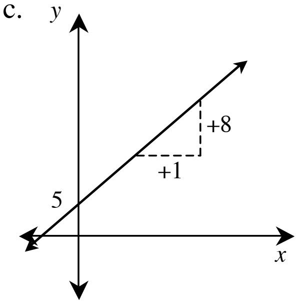
You have now used your knowledge of growth factors and Figure 0 to create tile patterns and $x \rightarrow y$ tables directly from rules. You have also looked at graphs to determine the equation or rule for the pattern. Today you will reverse that process and use an equation to create a graph without the intermediate step of creating an $x \rightarrow y$ table.

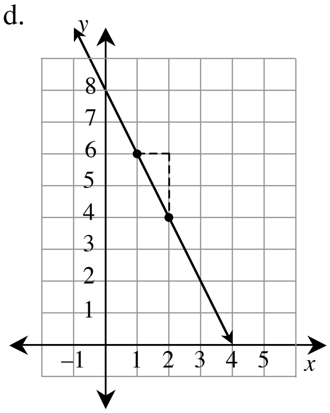
4-49. For each of the graphs below:

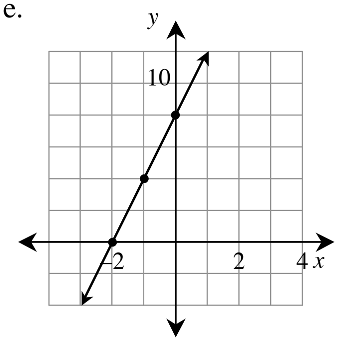
- Write a rule.
- Describe how the pattern changes and how many tiles are in Figure 0.

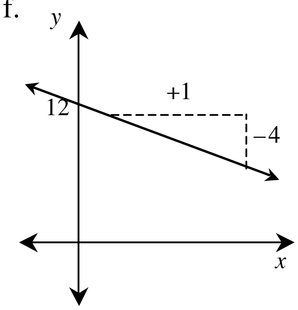
a.


b.


c.


d.


e.


f.


4-50. Now **reverse** the process. Graph the following rules without first making a table. Parts (a) and (b) can go on the same set of axes, as can parts (c) and (d). Label each line with its equation, ***y*-intercept** (where it crosses the *y*-axis), and a growth triangle.

a. $y = 4x + 3$

b. $y = 3x$

c. $y = -3x + 8$

d. $y = x - 1$

4-51. Sketch a graph that fits each description below and then label each line with its equation. You can put all of the graphs on one set of axes if you label the lines clearly. Use what you know about growth factor and Figure 0 to help you.

a. A pattern that has three tiles in Figure 0 and adds four tiles in each new figure.

b. A pattern that shrinks by three tiles between figures and starts with five tiles in Figure 0.

c. A pattern that has two tiles in all figures.

4-52. Now **reverse** your process to describe the pattern represented by the rule $y = -2x + 13$. Be as detailed as you can.

4-53. CONSOLIDATING YOUR LEARNING

a. Find the web that you updated at the end of Lesson 4.1.5. On it, add arrows for any new connections that you have made.

b. In your Learning Log, write a step-by-step process for **graphing directly from a rule**. A student who has not taken algebra yet should be able to read your process and understand how to create a graph. It may help you to think about these questions as you write:

What information do you get from your rule?

How does that information show up on the graph?

Where does your graph start?

How do you figure out the next point?

What should you label to make it a complete graph?

Title this entry "Graphing Without an $x \rightarrow y$ Table," and label it with today's date.

Review & Preview

4-54. Use what you know about m and b to graph each rule below without making a table. Draw a growth triangle for each line.

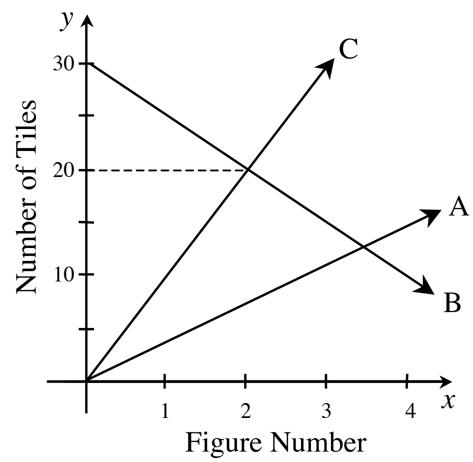
a. $y = 2x - 3$

b. $y = -2x + 5$

c. $y = 3x$

d. $y = \frac{1}{2}x + 1$

4-55. Examine the graph at right showing three tile patterns.



a. What do you know about Figure 0 for each of the three patterns?

b. Which pattern changes most quickly? How quickly does it change? Show how you know.

c. Which figure number has the same number of tiles in patterns B and C? Explain how you know.

d. Write a rule for pattern B.

4-56. Translate these algebraic statements into words: $y = 2x + 5$ and $y = 6x + 5$.

a. What do you know about Figure 0 for each pattern?

b. Which pattern grows most quickly? How do you know?

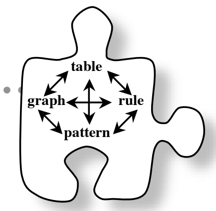
4-57. Evaluate each expression below when $x = -3$.

a. $4x + 16$

b. $3x^2 - 2x + 1$

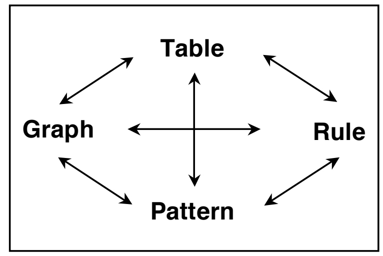
4-58. Ms. B is making snickerdoodle cookies. Her recipe uses one-and-a-half teaspoons of cinnamon to make two-dozen cookies. If she needs to make thirteen-dozen cookies in order to give one cookie to each of her students, how much cinnamon will she need?

4.1.7 What are the connections?



Completing the Web

After all of the work you have done with equations in $y = mx + b$ form, you know a lot about starting with one representation of a pattern and moving to different representations. Today you will work with your team to make sure you are confident moving around the representations web.

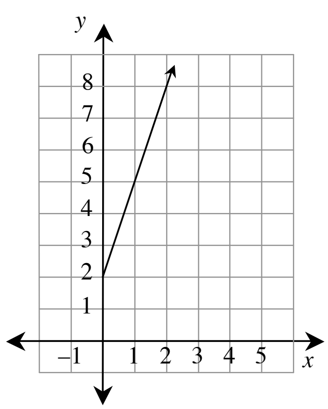


Answer problems 4-59 and 4-60 on graph paper. Discuss each problem with your team to get as much as you can out of these problems.

4-59. GRAPH $\rightarrow$ PATTERN and TABLE $\rightarrow$ PATTERN

On graph paper, draw tile patterns (Figures 0, 1, and 2) that could represent the data shown below. Be creative, but make sure that the growth of each pattern makes sense to your teammates.

a.



b.

x	***y***
0	14
1	11
2	8
3	5
4	2

4-60. REVISITING "GROWING, GROWING, GROWING"

Problem 1-32 from Chapter 1 asked you to determine which figure in the pattern shown below would have 79 tiles. Now that you know more about graphs, $x \rightarrow y$ tables, rules, and tile patterns themselves, you can show the answer to this question in multiple ways.

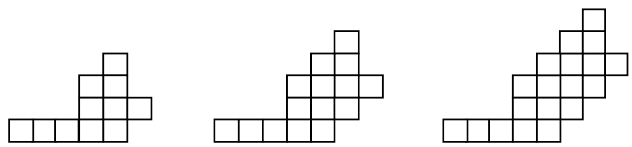
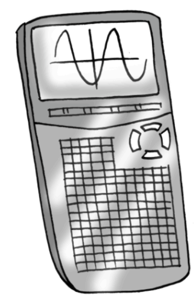


Figure 2 Figure 3 Figure 4



Your Task: Solve this problem by completing the following tasks. Use a graphing calculator or other graphing technology to help you find a graph and a table. Be sure to record your work and **justify** your thinking.

- Copy the three figures above onto a piece of graph paper. Extend the pattern on graph paper to include Figures 1 and 5.
- Find a rule, table, and graph for this pattern.
- Which figure will have 79 tiles? Use as many representations as you can to justify your answer.

4-61. EXTENSION

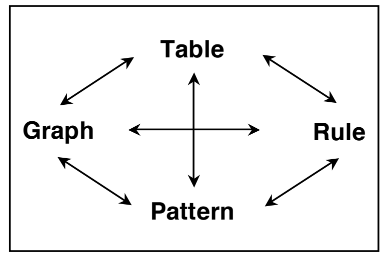
Invent an equation to fit these clues: The x-intercept is 2, and the pattern grows by 4. Show and explain your reasoning.

MATH NOTES

METHODS AND MEANINGS

Multiple Representations

Consider the areas of the figures in the **tile pattern** below. The number of tiles in each figure can also be represented in an ***x*→*y* table**, on a **graph**, or with a **rule** (equation).



Remember that in this course, tile patterns will be considered to be elements of continuous relationships and thus will be graphed with a continuous line or curve.

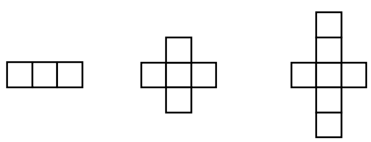
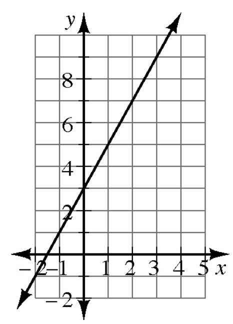


Figure 0 Figure 1 Figure 2

Tile Pattern



Graph

$$y = 2x + 3$$

Rule (Equation)

Figure Number (x)	0	1	2
Number of Tiles (y)	3	5	7

***x*→*y* Table**

Review & Preview

4-62. Use what you know about m and b to graph each equation below without making a table. Show a growth triangle on each graph and label the x- and y-intercepts.

a. $y = 3 - 2x$

b. $y = 2x$

c. $y = 3$

d. $y = -\frac{1}{2}x + 3$

4-63. Copy and complete each $x \rightarrow y$ table below on your paper. Using what you know about m and b, write an equation that represents the data in the table.

a.

x	y
0	5
1	7
2	9
3	11
4	13
30	
200	
	505
x	

b.

x	y
0	4
1	2
2	0
3	–2
4	–4
30	
150	
300	
x	

c.

x	y
–2	7
–1	4
0	1
1	–2
2	–5
3	
100	
	70
x	

4-64. Solve each of the following equations and check your answers.

a. $2x - 3 = 7$

b. $3x + 5 - x = x - 3$

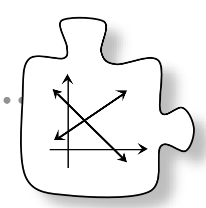
c. $5x - (x + 1) = 6 - 2x$

d. $0.5x - 5 = x + 4$

4-65. For a tile pattern with the rule $y = 6x + 4$ (where x represents the figure number and y represents the number of tiles), which figure number has 40 tiles in it? How do you know?

4-66. Josie and Jules are building a model car. They find that the real car is 54 inches tall and 180 inches long. They decide to make their model 3 inches tall, but now they are having a disagreement. Josie thinks that their model should be 10 inches long and Jules thinks it should be 129 inches long. Help them settle their argument by deciding if either of them is correct. Explain how you know exactly how long their model should be.

4.2.1 When are they the same?



Introduction to Systems of Equations

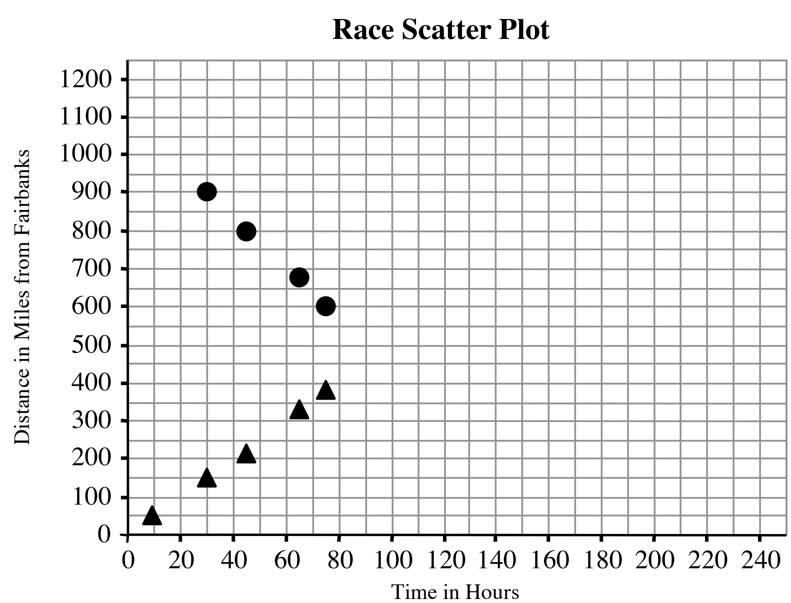
In Section 4.1, you graphed lines and curves that represented tile patterns. But what happens when you graph two lines at the same time? What can you learn? Today you will use data, graphs, and rules to examine what happens when two lines (or curves) intersect.

4-67. The Iditarod Trail Sled Dog Race is famous for its incredible length and its use of dogs. In 2003, the sled drivers, known as mushers, started their dog sleds at Fairbanks, Alaska and rode through the snow for many days until they reached Nome, Alaska. Along the route there were stations where the competitors checked in, so data was kept on the progress of each team.

Joyla and her team of dogs made it through the first five checkpoints. At the same time, her buddy Evie left Nome (the finish line) on the day the race started in an effort to meet Joyla and offer encouragement. Evie traveled along the route toward the racers on her snowmobile. The progress of each person is shown on the graph below.

Your Task: With your team, analyze the data in the graph. Consider the questions below as you work. Be prepared to defend your results.

- Which data represents Evie? Which represents Joyla? How can you tell?
- When did Evie meet Joyla?
- How long was the race? How can you tell?
- Who traveled faster? Explain how you know.
- Approximately how long did it take Joyla to finish the race? How did you find your answer?



4-68. The point where two lines (or curves) cross is called a **point of intersection**. Two or more lines (or curves) are called a **system of equations**. When you work with data, points of intersection can be meaningful, as you saw in the last problem.

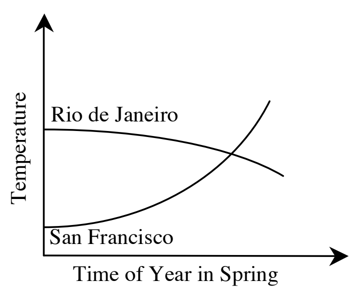
a. On graph paper, graph $y = 3x - 4$ and $y = -2x + 6$ on the same set of axes.

b. Find the point of intersection of these two lines and label the point with its coordinates; that is, write it in the form (x, y).

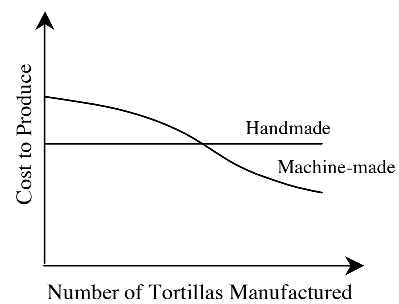
4-69. The meaning of a point of intersection depends on what the graph is describing. For example, in problem 4-67, the point where Joyla's and Evie's lines cross represents when they met during the race.

Examine each of the graphs below and write a brief story that describes the information in the graph. Include a sentence explaining what the point of intersection represents.

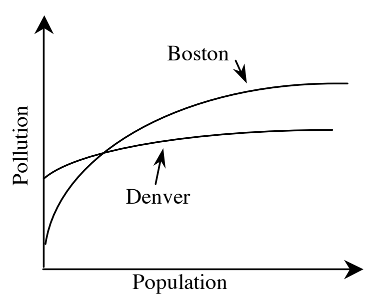
a.



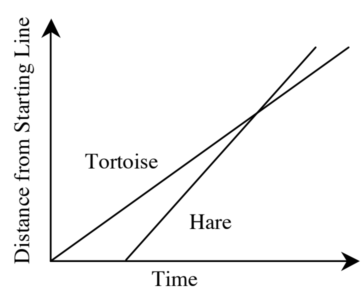
b.



c.



d.



4-70. In your Learning Log, write your own situation like the ones in problem 4-69 and make a graph. Have at least two lines or curves intersect. Explain what the intersection represents in your situation. Title this entry "Points of Intersection" and label it with today's date.

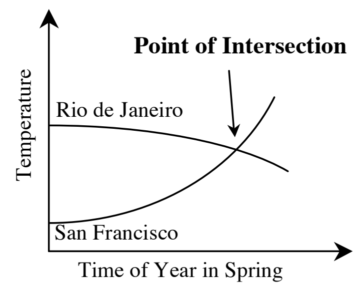
MATH NOTES

METHODS AND MEANINGS

Systems of Equations Vocabulary

The point where two lines (or curves) intersect is called a **point of intersection**. This point's significance depends on the context of the problem.

Two or more lines or curves used to find a point of intersection are called a **system of equations**. A system of equations can represent a variety of contexts and can be used to compare how two or more things are related. For example, the system of equations graphed at right compares the temperature in two different cities over time.

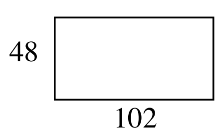


Review & Preview

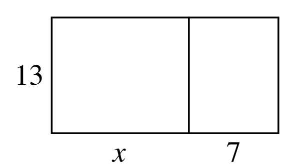
4-71. To ride to school, Elaine takes 15 minutes to ride 8 blocks. Assuming she rides at a constant speed, how long should it take her to go 20 blocks? Justify your answer.

4-72. Find the area of the entire rectangle in each diagram below. Show all work.

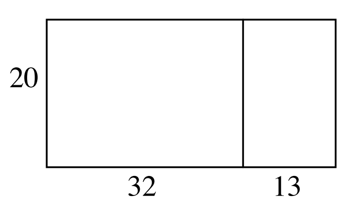
a.



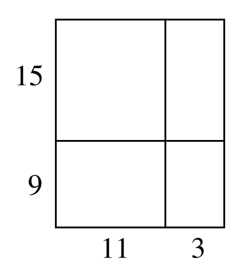
b.



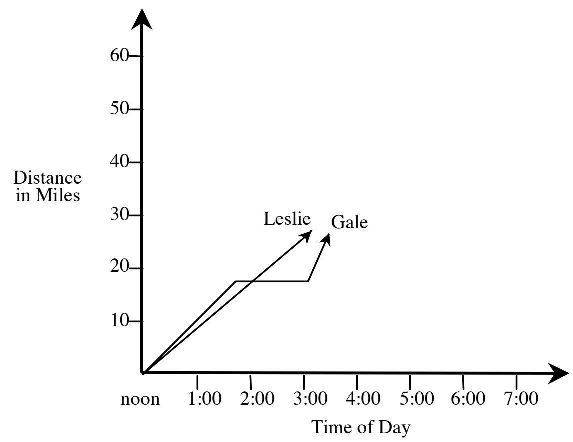
c.



d.



4-73. Gale and Leslie are engaged in a friendly 60-mile bike race that started at noon. The graph at right represents their progress so far.



a. What does the intersection of the two lines represent?

b. At what time (approximately) did Leslie pass Gale?

c. About how far had Leslie traveled when she passed Gale?

d. What do you think happened to Gale between 1:30 and 3:00?

e. If Leslie continues at a steady pace, when will she complete the race?

4-74. Write an equation (rule) for each of the $x \rightarrow y$ tables below. Then, on one set of axes, use each rule to graph.

a.

x	y
8	23
2	5
–3	–10
9	26
x	

b.

x	y
6	32
–2	–8
0	2
10	52
x	

4-75. Translate each part below from symbols into words or from words into symbols.

a. $-y+8$

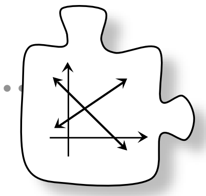
b. $2x-48$

c. $(x+3)^2$

d. The opposite of six times the square of a number.

e. A number multiplied by itself, then added to five.

4.2.2 When are they the same?



Writing Rules from Word Problems

In Lesson 4.2.1, you discovered that the point of intersection of two lines or curves can have an important meaning. Finding points of intersection is another strategy you can use to solve problems, especially those with two quantities being compared.

Analyze the following situations using the multiple tools you have studied so far.

4-76. BUYING BICYCLES

Latanya and George are saving up money because they both want to buy new bicycles. Latanya opened a savings account with $50. She just got a job and is determined to save an additional $30 a week. George started a savings account with $75. He is able to save $25 a week.

Your Task: Use at least **two different ways** to find the time (in weeks) when Latanya and George will have the same amount of money in their savings accounts. Be prepared to share your methods with the class.

4-77. Did you graph the scenario in problem 4-76? If not, graph a line for Latanya and another line for George on the same set of axes. Confirm your answer to problem 4-76 on the graph. Consider the questions below to help you decide how to set up the graph.

- What should the x-axis represent? What should the y-axis represent?
- How should the axes be scaled?
- Should the amounts in the savings accounts be graphed on the same set of axes or graphed separately? Why?

4-78. If you have not done so already, consider how to use rules to confirm the point of intersection for Latanya's and George's lines.

a. Write a rule for Latanya's savings account.

b. Write a rule for George's savings account.

c. Use the rules to check your solution to problem 4-76.

4-79. Gerardo decided to use tables to find the point of intersection of the lines $y = 4x - 6$ and $y = -2x + 3$. His tables are shown below.

$y = 4x - 6$

IN (x)	–3	–2	–1	0	1	2	3
OUT (y)	–18	–14	–10	–6	–2	2	6

$y = -2x + 3$

IN (x)	–3	–2	–1	0	1	2	3
OUT (y)	9	7	5	3	1	–1	–3

a. Examine his tables. Is there a common point that makes both rules true? If not, can you describe where the point of intersection is?

b. Now graph the rules on the same set of axes. Where do the lines intersect?

c. Use the rules to confirm your answer to part (b).

Review & Preview

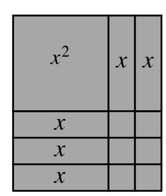
4-80. It's the end of the semester, and the clubs at school are recording their profits. The Science Club started out with \$20 and has increased its balance by an average of \$10 per week. The Math Club saved \$5 per week and started out with \$50 at the beginning of the semester.

a. Create an equation for each club. Let x represent the number of weeks and y represent the balance of the club's account.

b. Graph both lines on one set of axes. When do the clubs have the same balance?

c. What is the balance at that point?

4-81. Examine the rectangle formed with algebra tiles at right.

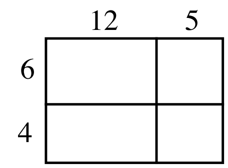


a. Find the area of the entire rectangle. That is, what is the sum of the areas of the algebra tiles?

b. Find the perimeter of the entire rectangle. Show all work.

4-82. On graph paper, plot the points (–3, 7) and (2, –3) and draw a line through them. Then name the x- and y-intercepts of the line.

4-83. Use the rectangle at right to answer the following questions.

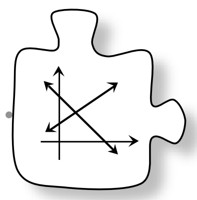


a. Find the area of the entire rectangle. Explain how you found your solution.

b. Calculate the perimeter of the figure.

4-84. In Spring, the daily high temperature in Boulder, Colorado rises about $\frac{1}{3}$ degree per day. On Friday, May 2, the temperature reached 74°. Predict when the temperature will reach 90°.

4.2.3 When are they the same?



Solving Systems Algebraically

So far in Section 4.2, you have solved systems of equations by graphing two lines and finding where they intersect. However, it is not always convenient (nor accurate) to solve by graphing.

Today you will explore a new way to approach solving a system of equations. Questions to ask your teammates today include:

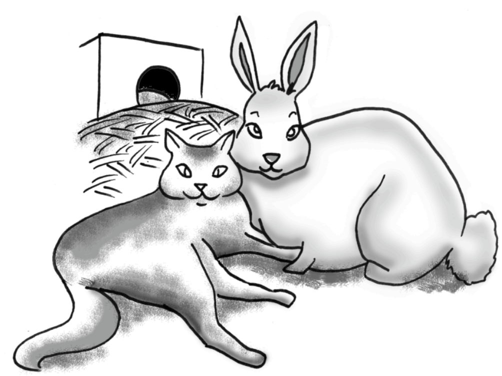
How can you find a rule?

How can you compare two rules?

How can you use what you know about solving?

4-85. CHUBBY BUNNY

Use tables, rules, and a graph to find and check the solution for the problem below.



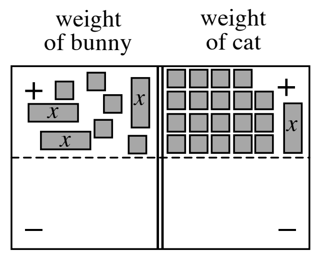
Barbara has a bunny that weighs 5 pounds and gains 3 pounds per year. Her cat weighs 19 pounds and gains 1 pound per year. When will the bunny and the cat weigh the same amount?

4-86. SOLVING SYSTEMS OF EQUATIONS ALGEBRAICALLY

In problem 4-85, you found rules like those shown below to represent the weights of Barbara's cat and bunny. For these rules, x represents the number of years and y represents the weight of the animal.

$$y = \underbrace{5+3x}_{\text{weight of bunny}} \quad \text{and} \quad y = \underbrace{19+x}_{\text{weight of cat}}$$

Since you want to know when the weights of the cat and bunny are the same, you can use an equation mat to represent this relationship, as shown at right.



a. Problem 4-85 asked you to determine when the weight of the cat and the bunny are the same. Therefore, you want to determine when the expressions on the left (for the bunny) and the right (for the cat) are equal. Write an equation that represents this balance.

b. Solve your equation for x, which represents years. According to your solution, how many years will it take for the bunny and the cat to weigh the same number of pounds? Does this answer match your answer from the graph of problem
4-85?

c. How much do the cat and bunny weigh at this time?

4-87. CHANGING POPULATIONS

Post Falls High School in Idaho has 1160 students and is growing by 22 students per year. Richmond High School in Indiana has 1900 students and is shrinking by 15 students per year.

a. Without graphing, write a rule that represents the population at Richmond High School and another rule that represents the population at Post Falls High School. Let x represent years and y represent population.

b. Graphing the rules for part (a) is challenging because of the large numbers involved. Using a table could take a long time. Therefore, this problem is a good one to solve algebraically, the way you solved problem 4-86.

Use the rules together to write an equation that represents when these high schools will have the same population. Then solve your equation to find out when the schools' populations will be the same.

c. What will the population be at that time?

4-88. PUTTING IT ALL TOGETHER

Find the solution to the problem below by **graphing** and also by **solving an equation**. The solutions using both methods should match, so be sure to review your work carefully if the results disagree.

Your school planted two trees when it was first opened. One tree, a ficus, was 6 feet tall when it was planted and has grown 1.5 feet per year. The other tree, an oak, was grown from an acorn (on the ground) and has grown 2 feet per year. When will the trees be the same height? How tall will the trees be when they are the same height?

4-89. Ms. Harlow calls the method you have been using today to solve equations the **Equal Values Method**. Explain why this name makes sense.

MATH NOTES

METHODS AND MEANINGS

Solving a Linear Equation

When solving an equation like the one shown below, several important strategies are involved.

- **Simplify.** Combine like terms and "make zeros" on each side of the equation whenever possible.
- **Keep equations balanced.** The "equals" sign in an equation indicates that the expressions on the left and right are balanced. Anything done to the equation must keep that balance.
- **Get *x* alone.** Isolate the variable on one side of the equation and the constants on the other.

$$\begin{array}{rcll}
3x-2+4 &=& x-6 & \text{combine like terms} \\
3x+2 &=& x-6 & \\
\underline{-x} &=& \underline{-x} & \text{subtract } x \text{ on both sides} \\
2x+2 &=& -6 & \\
\underline{-2} &=& \underline{-2} & \text{subtract 2 on both sides} \\
\dfrac{2x}{2} &=& \dfrac{-8}{2} & \text{divide both sides by 2} \\
x &=& -4 &
\end{array}$$

- **Undo operations.** Use the fact that addition is the opposite of subtraction and that multiplication is the opposite of division to solve for *x*. For example, in the equation $2x = -8$, since the 2 and *x* are multiplied, dividing both sides by 2 will get *x* alone.

Review & Preview

4-90. A local restaurant offers a Dim Sum lunch special that includes two dumplings, three egg rolls, a sweet bun, and a drink. Susan and her friends ordered four Dim Sum lunch specials.

How many of each item should they receive?

4-91. Kenneth claims that (2, 0) is the point of intersection of the lines $y=-2x+4$ and $y=x-2$. Is he correct? How do you know?

4-92. Graph the lines $y=2x-3$ and $y=-x+3$.

a. Where do they intersect? Label the point on the graph.

b. Find the point of intersection using the Equal Values Method. That is, start by combining both equations into one equation that you can solve for *x*.

c. Which method is easier, graphing or using algebra to solve?

4-93. Determine the coordinates of each point of intersection without graphing.

a. $y=2x-3$
$y=4x+1$

b. $y=2x-5$
$y=-4x-2$

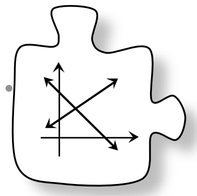
4-94. MORE OR LESS

Judy has \$20 and is saving at a rate of \$6 per week. Ida has \$172 and is spending at a rate of \$4 per week. After how many weeks will each have the same amount of money?

a. Write an equation using *x* and *y* for Judy and Ida. What does *x* represent? What does *y* represent?

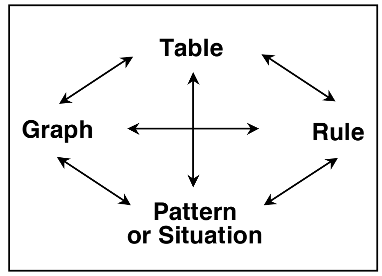
b. Solve this problem using any method you choose.

4.2.4 How can I use $y = mx + b$?



Extending the Web to Other Linear Situations

Today you will take what you have learned in this chapter and **apply** it to linear situations that are not tile patterns.



4-95. EXTENDING THE WEB TO NEW SITUATIONS: TEAM CHALLENGE

Today you are going to **apply** what you know about the starting point (Figure 0), growth factor, and the **connections** between representations to answer some challenging questions in real-life situations. The information in each question, parts (a) through (e) below, describes a different situation. All of the situations are **linear** (when graphed, they are lines).

Based on the given information, answer the questions in each problem. Show your answers completely and explain your strategies for answering the questions. You may answer these problems in any order, but make sure you answer each one completely before moving to another problem.

Work together as a team. The more you listen to how other people see the **connections** and share your own ideas, the more you will know at the end of this challenge. Stick together and be sure to talk through every idea.

Each person will turn in his or her own paper at the end of activity, showing solutions and explanations to each problem. Your work does *not* need to be identical to your teammates' work, but you should have talked and agreed that all explanations are correct.

a. SAVING MONEY

Julia has $325 in her savings account. She just got a new job and will be saving money every month. If she always deposits the same amount, how much money will be in her account after she has been saving for a year? (Assume she never spends money from this account.)

Number of Months	Money in Account
…	…
7	$780
8	$845
9	$910
…	…

Problem continues on next page →

4-95. *Problem continued from previous page.*

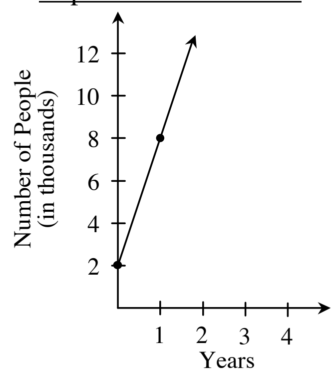
b. POPULATION GROWTH

The $x \rightarrow y$ table, graph, rule, and words below each describe a different town. Based on the information you are given about each town's population, decide which town is growing the fastest. Explain how you know.

Population of Town A

Year	Number of People
1975	32,000
1979	50,000
1980	54,500

Population of Town B



Population of Town C

If x = year and y = number of people, then:

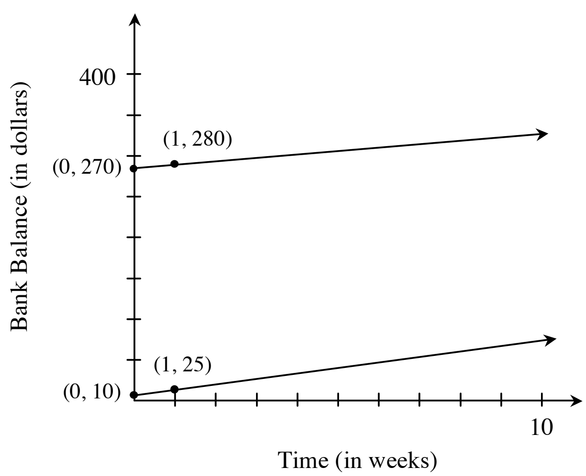
$$y = 46000 - 5200x$$

Population of Town D

Town D is growing. Oddly, the same number of people moves to the town each year. Two years ago, the town had 9100 people. Now the town has 15,500 people.

c. FUNDRAISING

The graph at right describes the money two clubs are earning from fundraising. In how many weeks will the two clubs have the same amount of money? Explain your thinking completely.

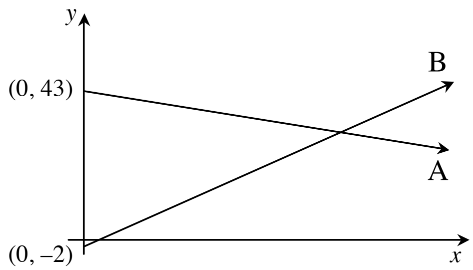


Problem continues on next page →

4-95. *Problem continued from previous page.*

d. STORY TIME

The graph and $x \rightarrow y$ tables below describe a situation. Write a story that fits the given information. Show the **connections** between the information you are given and the information in your story. Your story must give meaning to the point of intersection.



Line A

x	y
⋮	⋮
8	11
9	7
10	3
⋮	⋮

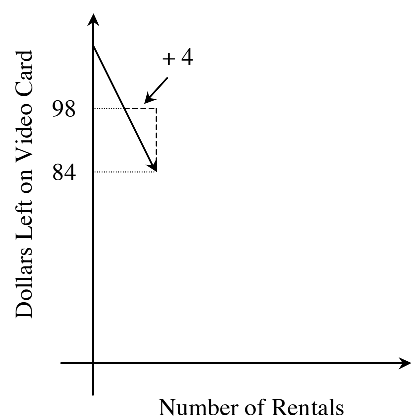
Line B

x	y
⋮	⋮
4	22
5	28
⋮	⋮

e. VIDEO RENTAL

Gina has a prepaid video rental card. She currently has a credit of $84 on the card.

The graph at right describes the amount of money there was on the card recently. Use this information to determine:



- How much one video rental costs.
- How many more videos can Gina rent before the card is used up.

METHODS AND MEANINGS

MATH NOTES

The Equal Values Method

The **Equal Values Method** is a non-graphing method to find the point of intersection or solution to a system of equations.

Start with two equations in $y = mx + b$ form, such as $y = -2x + 5$ and $y = x - 1$. Take the two expressions that equal y and set them equal to each other. Then solve this new equation to find x. See the example at right.

$$\begin{gathered} -2x + 5 = x - 1 \\ -3x = -6 \\ x = 2 \end{gathered}$$

Once you know the x-coordinate of the point of intersection, substitute your solution for x into *either* original equation to find y. In this example, the first equation is used.

$$\begin{gathered} y = -2x + 5 \\ y = -2(2) + 5 \\ y = 1 \end{gathered}$$

A good way to check your solution is to substitute your solution for x into *both* equations to verify that you get equal y-values.

$$\begin{gathered} y = x - 1 \\ y = (2) - 1 \\ y = 1 \end{gathered}$$

Write the solution as an ordered pair to represent the point on the graph where the equations intersect.

(2, 1)

Review & Preview

4-96. Ariyonne claims that (3, 6) is the point of intersection of the lines $y = 4x - 2$ and $y = \frac{1}{2}x + 5$. Is she correct? How do you know?

4-97. Determine the coordinates of each point of intersection without graphing.

a. $y = -x + 8$
$y = x - 2$

b. $y = -3x$
$y = -4x + 2$

4-98. Graph the lines $y = 2x - 3$ and $y = 2x + 1$.

a. Where do they intersect?

b. Solve this system using the Equal Values Method.

c. Explain how your graph and algebraic solution relate to each other.

4-99. CHANGING POPULATIONS

Highland has a population of 12,200. Its population has been increasing at a rate of 300 people per year. Lowville has a population of 21,000 but is declining by 250 people per year. Assuming these rates do not change, in how many years will the populations be equal?

a. Write an equation that represents each city's population over time. What do your variables represent?

b. Solve the problem. Show your work.

4-100. The table below shows the amount of money Francis had in his bank account each day since he started his new job.

Days at New Job	Money in Account
0	\$27
1	\$70
2	\$113
3	\$156

a. Write a rule for the amount of money in Francis's account. Let *x* represent the number of days and *y* represent the number of dollars in the account.

b. When will Francis have more than \$1000 in his account?

4-101. Kathy is thinking of a number. When she triples her number, adds eighteen, and then subtracts her original number from the sum, she gets four. What is Kathy's original number?

4-102. Graph the equation $y = -2x^2 - 4x$. Start by making an $x \rightarrow y$ table. Be sure to include negative values for x.

4-103. Solve this problem using Guess and Check. Write your solution in a sentence.

The number of students attending the Fall play was 150 fewer than three times the number of adults. Together, students and adults purchased 1778 tickets. How many students attended the Fall play?

4-104. Predict where each rule will cross the y-axis.

a. $y = 17x + 3$

b. $y = \frac{16}{3}x - \frac{5}{12}$

c. $y = 12 - 4x$

4-105. When Ellen started with Regina's favorite number and tripled it, the result was twelve more than twice the favorite number. Define a variable, write an equation, and then use the equation to find Regina's favorite number.

4-106. Graph the lines $y = -4x - 3$ and $y = -4x + 1$ on graph paper.

a. Where do they intersect?

b. Solve this system using the Equal Values Method.

c. Explain how your graph and algebraic solution relate to each other.

Chapter 4 Closure   What have I learned?

Reflection and Synthesis

The activities below offer you a chance to reflect on what you have learned during this chapter. As you work, look for concepts that you feel very comfortable with, ideas that you would like to learn more about, and topics you need more help with. Look for **connections** between ideas as well as **connections** with material you learned previously.

① TEAM BRAINSTORM

With your team, brainstorm a list for each of the following topics. Be as detailed as you can. How long can you make your list? Challenge yourselves. Be prepared to share your team's ideas with the class.

Topics: What have you studied in this chapter? What ideas and words were important in what you learned? Remember to be as detailed as you can.

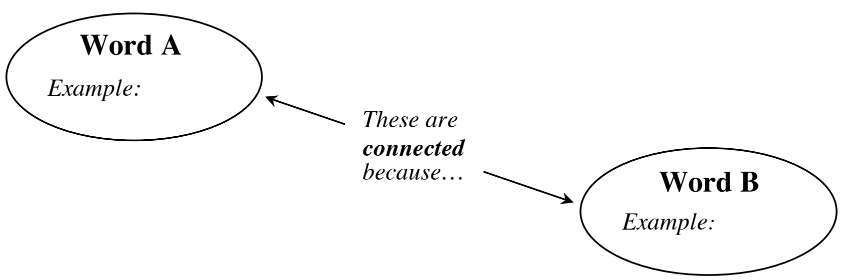
Connections: What topics, ideas, and words that you learned *before* this chapter are **connected** to the new ideas in this chapter? Again, make your list as long as you can.

② MAKING CONNECTIONS

The following is a list of the vocabulary used in this chapter. The words that appear in bold are new to this chapter. Make sure that you are familiar with all of these words and know what they mean. Refer to the glossary or index for any words that you do not yet understand.

b	continuous	coordinates
dependent variable	discrete	**Equal Values Method**
equation	**Figure 0**	graph
growth	independent variable	***m***
pattern	**point of intersection**	**representation**
rule	solution	starting value
system of equations	**web**	$x \rightarrow y$ table
x- and y-intercepts	$\boldsymbol{y = mx + b}$	

Make a concept map showing all of the **connections** you can find among the key words and ideas listed above. To show a **connection** between two words, draw a line between them and explain the **connection**, as shown in the example below. A word can be **connected** to any other word as long as there is a **justified connection**. For each key word or idea, provide a sketch that illustrates the idea (see the example below).



Your teacher may provide you with vocabulary cards to help you get started. If you use the cards to plan your concept map, be sure either to re-draw your concept map on your paper or to glue the vocabulary cards to a poster with all of the **connections** explained for others to see and understand.

While you are making your map, your team may think of related words or ideas that are not listed above. Be sure to include these ideas on your concept map.

③ SUMMARIZING MY UNDERSTANDING

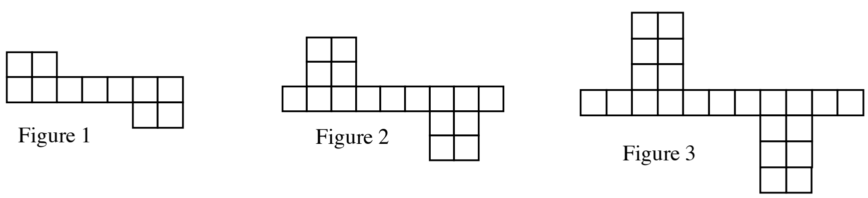
This section gives you an opportunity to show what you know about certain math topics or ideas. Your teacher will give you directions for exactly how to do this.

④ WHAT HAVE I LEARNED?

This section will help you evaluate which types of problems you have seen with which you feel comfortable and those with which you need more help. This section appears at the end of every chapter to help you check your understanding. Even if your teacher does not assign this section, it is a good idea to try the problems and find out for yourself what you know and what you need to work on.

Solve each problem as completely as you can. The table at the end of the closure section has answers to these problems. It also tells you where you can find additional help and practice on problems like these.

CL 4-107. Examine the pattern below.



a. On graph paper, sketch Figure 0 and Figure 4.

b. Make a table showing Figure 0 through Figure 4.

c. Write a rule to represent the pattern.

d. On graph paper, create a graph of the number of tiles in each figure.

e. What is the growth for the pattern?

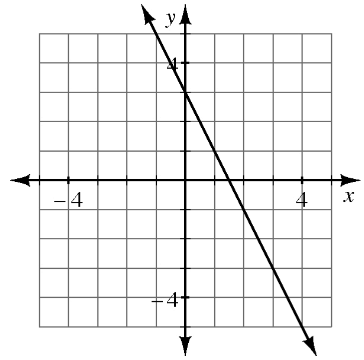
f. Predict how many tiles Figure 100 will have.

CL 4-108. Are the two expressions below equal? Show how you know.

$$4x^2 + 2x - 5 - 3x \quad \text{and} \quad 6x^2 - x + 3 - 2x^2 - 8$$

CL 4-109. Priscilla and Ursula went fishing. Priscilla brought a full box of 32 worms and used one worm every minute. Ursula brought a box with five worms and decided to dig for more before she began fishing. Ursula dug up two worms per minute. When did Priscilla and Ursula have the same number of worms? Show how you know.

CL 4-110. Examine the graph at right.



a. Give two ways you can tell that the rule $y = 2x - 3$ does not match the graph.

b. Make a graph that matches the rule $y = 2x - 3$.

c. Find a rule that represents the graph at right.

CL 4-111. Consider the rule $y = 5x + 7$.

a. How many tiles are in Figure 0?

b. Which figure has 37 tiles?

c. In the equation $y = mx + b$, what do the letters m and b represent?

CL 4-112. For each pair of lines below, solve the system by **graphing** and solve it **algebraically** using the Equal Values Method. Explain how the graph confirms the algebraic result.

a. $y = 7x - 5$ and $y = -2x + 13$

b. $y = 3x - 1$ and $y = 3x + 2$

CL 4-113. To rent a jet ski at Sam's costs \$25 plus \$3 per hour. At Claire's, it costs \$5 plus \$8 per hour. At how many hours will the rental cost at both shops be equal?

a. Write an equation that represents each shop's charges. What do your variables represent?

b. Solve the problem. Show your work.

CL 4-114. Simplify the following expressions, if possible.

a. $x + 4x - 3 + 3x^2 - 2x$

b. $2x + 4y^2 - 6y^2 - 9 - x + 3x$

c. $3x^2 + 10y - 2y^2 + 4x - 14$

d. $20 + 3xy - 4xy + y^2 + 10 - y^2$

e. Evaluate the expressions in parts (a) and (b) above when $x = 5$ and $y = -2$.

CL 4-115. Copy and complete the table for the linear pattern below.

IN (x)	–4	–3	–2	–1	0	1	2	3	4
OUT (y)					–2	3	8		

a. What is the y-intercept? What is the growth factor?

b. Find the rule for this line.

c. If the output number (y) is –52, what was the input number (x)?

CL 4-116. Use Guess and Check to solve the problem below.

For the school play, the advance tickets cost \$3 and tickets at the door cost \$5. Thirty more tickets were sold at door than in advance, and \$2630 was collected. How many tickets were sold at the door? Write your answer in a sentence.

CL 4-117. Check your answers using the table at the end of the closure section. Which problems do you feel confident about? Which problems were hard? Use the table to make a list of topics you need help on and a list of topics you need to practice more.

⑤ HOW AM I THINKING?

This course focuses on five different **Ways of Thinking**: reversing thinking, justifying, generalizing, making connections, and applying and extending understanding. These are some of the ways in which you think while trying to make sense of a concept or to solve a problem (even outside of math class). During this chapter, you have probably used each Way of Thinking multiple times without even realizing it!

This closure activity will focus on one of these Ways of Thinking: **reversing thinking**. Read the description of this Way of Thinking at right.

Think about the topics that you have learned during this chapter. When did you undo a process? When did you try to go backward in your problem-solving process? You may want to flip through the chapter to refresh your memory about the problems that you have worked on. Discuss any ideas you have with the rest of the class. Once your discussion is complete, examine some of the ways you have **reversed your thinking** as you answer the questions below.

Reversing Thinking

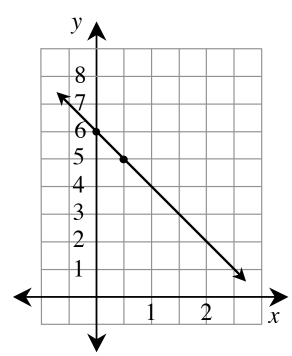
To reverse your thinking can be described as "thinking backward." You think this way when you want to understand a concept in a new direction. Often, it requires you to try to undo a process. When you catch yourself thinking, "*What if I try to go backwards?*", you are reversing your thinking.

a. If you know how to go from one representation to another, then there is a way to **reverse the process**. Consider the web connection graph ↔ rule.

   i. Find the equation of the line graphed at right.

   ii. Now **reverse the process**. On graph paper, graph the rule $y = 2x - 3$.

   iii. Explore another connection on the web where you **reversed your thinking**. Find or create a problem that represents one direction of solving. Then write and solve another problem that requires you to reverse the process.



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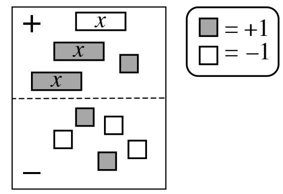
⑤ *Continued from previous page.*

b. Usually, if you change an expression into an equivalent expression, there is a way to **reverse the process** to return to the original expression. Consider this as you answer the questions below.



i. On your paper, write and simplify the expression represented on the expression mat at right.

ii. Draw an expression mat on your paper that uses 8 tiles and has a value of $x+3$.

iii. Explain how you **reversed your thinking** from part (*i*) to solve part (*ii*) above.

c. Now consider how you can **reverse your thinking** when solving a problem with proportional relationships.

i. For example, if Mr. Wallis pays $25 for 10 gallons of gasoline, how much would he pay to fill his scooter (which uses 3 gallons of gasoline)?

ii. Now write a question about Mr. Wallis's gasoline use that would require you to **reverse your thinking** from part (a) to solve. Explain why and how it requires you to **reverse your thinking**.

Answers and Support for Closure Activity #4
What Have I Learned?

Problem	Solution	Need Help?	More Practice
CL 4-107.	a. (see figures below)	Sections 3.1 and 4.1, Lesson 4.1.7 Math Notes box	Problems 4-8, 4-9, 4-11, 4-21, 4-35, 4-36, 4-37, 4-49, 4-59, and 4-60

a.

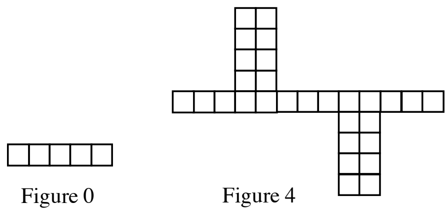


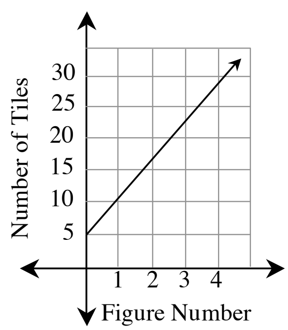
Figure 0   Figure 4

b.

Figure Number	0	1	2	3	4
Number of Tiles	5	11	17	23	29

c. $y = 6x + 5$

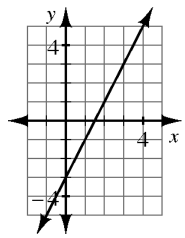
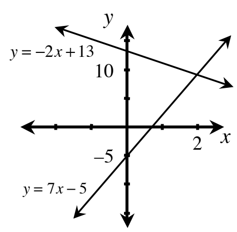
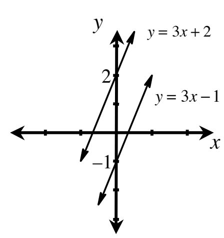
d.



e. Each figure has 6 more tiles than the previous figure.

f. Figure 100 will have 605 tiles.

Problem	Solution	Need Help?	More Practice
CL 4-108.	yes; $4x^2 - x - 5 = 4x^2 - x - 5$	Lesson 2.1.5 Math Notes box	Problem 4-6
CL 4-109.	They will have the same number of worms after 9 minutes.	Lesson 4.2.1 Math Notes box, Lesson 4.2.2	Problems 4-80, 4-85, 4-87, 4-88, 4-94, and 4-99

Problem	Solution	Need Help?	More Practice
CL 4-110.	a. The line goes down as x increases. The y-intercept is at +3. b. See graph at right. c. $y = -2x + 3$ 	Lessons 4.1.3, 4.1.4, and 4.1.6; Lesson 4.1.7 Math Notes box	Problems 4-19, 4-27, 4-36, 4-49, 4-50, 4-51, 4-54, and 4-55
CL 4-111.	a. There are 7 tiles in figure 0. b. Figure 6 has 37 tiles. c. m represents the growth factor, and b represents the number of tiles in Figure 0.	Lessons 4.1.2 and 4.1.4, Lesson 4.1.7 Math Notes box	Problems 4-10, 4-11, 4-26, 4-29, 4-52, 4-54, 4-56, and 4-62
CL 4-112.	a. The two lines intersect at the point (2, 9).  b. There is no solution to the system of equations, because the lines are parallel. 	Lesson 4.2.1 Math Notes box, Lesson 4.2.3, problem 4-86, Lesson 4.2.4 Math Notes box	Problems 4-68, 4-88, 4-92, 4-96, 4-97, 4-98, and 4-106
CL 4-113.	a. $y = 25 + 3x$ and $y = 5 + 8x$; x represents the number of hours rental, and y represents the cost. b. After 4 hours of ski rental, the cost at both shops will be equal.	Lesson 4.2.1 Math Notes box, Lesson 4.2.2	Problems 4-80, 4-85, 4-87, 4-88, 4-94, and 4-99

Problem	Solution	Need Help?	More Practice
CL 4-114.	a. $3x^2+3x-3$ b. $-2y^2+4x-9$ c. It cannot be simplified any further. d. $-xy+30$ e. (a) 87; (b) 3	Lesson 2.1.3 Math Notes box, Lesson 2.1.5 Math Notes box	Problems 4-6 and 4-57
CL 4-115.	See table below. a. The starting value is –2. The growth factor is 5. b. $y=5x-2$ c. –10	Lesson 4.1.4, Lesson 4.1.7 Math Notes box	Problems 4-7, 4-28, 4-34, 4-35, 4-63, 4-74, and 4-100
CL 4-116.	340 tickets were sold at the door.	Problems 1-41 and 1-42, Lesson 2.1.7 Math Notes box	Problems 4-24 and 4-103

CL 4-115 table:

IN (x)	–4	–3	–2	–1	0
OUT (y)	–22	–17	–12	–7	–2

table continued:

1	2	3	4
3	8	13	18